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Superinflation in General Relativity and Brans–Dicke Theories: An Eternal Universe?

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An eternal and always classical picture follows from "superinflationary" models (as defined by Pimentel for Brans–Dicke cosmology with $\Lambda \neq 0$); we study general relativistic and Brans–Dicke (with $\Lambda = 0$) cosmological models.

1. INTRODUCTION

Pimentel (1989) proposed, for a Brans and Dicke (1961) framework with a cosmological constant (Uehara and Kim, 1982), what he called "superinflation," namely

$$R = R_0 e^{\alpha t^2} \qquad (R_0,\ \alpha \text{ constants}) \tag{1}$$

Robertson–Walker (RW) metric, where R stands for the "radius" of the universe.

We show that in general relativity a similar theory can be proposed; it has all the qualities that have made inflationary models acceptable, and it also rivals the steady-state theory of Hoyle (1948) and Bondi and Gold (1948). We also show that superinflation in Brans–Dicke (BD) theory can also occur without a cosmological constant. We suggest that the law (1) could point to an "eternal" universe.

2. SUPERINFLATION IN GENERAL RELATIVITY

Let us consider a flat universe in general relativity, with a cosmological constant and RW metric. The field equations are (Weinberg, 1972)

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$$K\rho = 3H^2 - \Lambda \tag{2}$$

$$Kp = -2\ddot{R}R^{-1} - H^2 + \Lambda \tag{3}$$

where K, ρ, H, Λ, and p stand, respectively, for Einstein's gravitational constant, the energy density, Hubble's parameter, the cosmological constant, and the cosmic pressure.

An easy calculation shows that

$$H = 2\alpha t \equiv \frac{\dot{R}}{R} \tag{4}$$

$$K\rho = 12\alpha^2 t^2 - \Lambda \tag{5}$$

$$K\rho = -12\alpha^2 t^2 - 4\alpha + \Lambda \tag{6}$$

$$p = -\rho - \frac{4\alpha}{K} \tag{7}$$

Defining, as usual, the deceleration parameter as

$$q = -\frac{\ddot{R}R}{\dot{R}^2} \tag{8}$$

we find

$$\lim_{t\to\infty} q = -1 \tag{9}$$

Equation (7) is very similar to the violated energy condition $p = -\rho$ of the usual inflationary state. However, neither p nor ρ is constant in (7), so that we have a different situation from that of a "perfect cosmological principle" of steady-state theory.

We suggest a physical interpretation for the constant R_0 as the Planck length. This universe has no beginning, no end, and is always classical.

If $\Lambda < 0$, we have the condition of positivity of energy,

$$\rho \geq 0 \tag{10}$$

Mass creation can be easily calculated, and does not contradict available experimental data, being undetectable by present technology (Narlikar, 1983).

3. SUPERINFLATION IN BRANS–DICKE THEORY

We shall now show that in BD theory we can have superinflation without Λ. Pimentel (1989) says that he found an inflationary solution including the case $p = -\rho$. However, a close examination shows that in Pimentel's calculation, we obtain in this case $\dot{G} = 0$, which means that this is no longer a

BD theory. [The correct calculation is done in Berman (1989).] Pimentel's superinflation requires a law

$$R = R_0 e^{(\Lambda/a)t^2} \tag{11}$$

where Λ is the cosmological constant and a = const.

In this case $\Lambda = 0$ implies a static universe.

We shall now show that, with law (1), we can find superinflation with $\Lambda = 0$.

Weinberg (1972) finds BD field equations for a Robertson–Walker metric (and a perfect fluid):

$$\frac{d}{dt}(\dot{\phi}R^3) = \frac{8\pi}{3 + 2\omega}(\rho - 3p)R^3 \tag{12}$$

$$\dot{\rho} = -3H(\rho + p) \tag{13}$$

$$H^2 + kR^{-2} = \frac{8\pi\rho}{3\phi} - \dot{\phi}\phi^{-1}H + \frac{\omega}{6}\dot{\phi}^2\phi^{-2} \tag{14}$$

Returning to the flat case ($k = 0$), with ϕ representing an inverse gravitational "constant" (the scalar field) and ω the coupling constant, we find, for law(1),

$$\rho = AMt^2 e^{\beta t^2} \qquad (A, M, \text{ constants}) \tag{15}$$

$$\phi = -Me^{\beta t^2} \qquad (\beta \text{ constant}) \tag{16}$$

$$p = -AMe^{\beta t^2}\{\alpha t^2 + (1 + \beta t^2)(3\alpha)^{-1}\} \tag{17}$$

We find that

$$\omega \approx -20 \tag{18}$$

$$-\alpha = \beta = 0.2 \tag{19}$$

$$A = 5\pi^{-1}\beta^2 \tag{20}$$

4. FINAL COMMENTS

Superinflation is just a possibility. Either as a substitute for inflation, or as the model for an eternal universe, we think that it merits more study in the future. In a certain sense, additional BD structure with $\Lambda = 0$ is equivalent to Einstein's theory with $\Lambda \neq 0$. We have no initial singularity in these models. Because ω has a strange value in our calculation, we may prefer a $\Lambda \neq 0$ model in the BD framework, as studied by Pimentel.

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